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Analytic treatment of the excited instability spectra of the magnetically charged SU(2) Reissner-Nordström black holes

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Abstract: The magnetically charged SU(2) Reissner-Nordström black-hole solutions of the coupled nonlinear Einstein-Yang-Mills field equations are known to be characterized by infinite spectra of unstable (imaginary) resonances $\{\omega_n(r_+,r_-)\}_{n=0}^{n=\infty}$ (here $r_\pm$ are the black-hole horizon radii). Based on direct *numerical* computations of the black-hole instability spectra, it has recently been observed that the excited instability eigenvalues of the magnetically charged black holes exhibit a simple universal behavior. In particular, it was shown that the numerically computed instability eigenvalues of the magnetically charged black holes are characterized by the small frequency universal relation $\omega_n(r_+-r_-)=\lambda_n$, where $\{\lambda_n\}$ are dimensionless constants which are independent of the black-hole parameters. In the present paper we study analytically the instability spectra of the magnetically charged SU(2) Reissner-Nordström black holes. In particular, we provide a rigorous *analytical* proof for the *numerically*-suggested universal behavior $\omega_n(r_+-r_-)=\lambda_n$ in the small frequency $\omega_n r_+ \ll (r_+-r_-)/r_+$ regime. Interestingly, it is shown that the excited black-hole resonances are characterized by the simple universal relation $\omega_{n+1}/\omega_n = e^{-2\pi/\sqrt{3}}$. Finally, we confirm our analytical results for the black-hole instability spectra with numerical computations.

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Contents

1 Introduction

It is well known that the electrically charged U(1) Reissner-Nordström black-hole spacetime describes a *stable* solution of the coupled nonlinear Einstein-Maxwell field equations [1, 2] (see also [3–5]). On the other hand, the magnetically charged SU(2) Reissner-Nordström black-hole spacetime [6] describes an *unstable* solution of the coupled nonlinear Einstein-Yang-Mills field equations [7–13]. In fact, the Reissner-Nordström black-hole solutions of the Einstein-Yang-Mills theory are characterized by an *infinite* spectrum of unstable perturbation modes. These unstable (exponentially growing in time) modes are described by an infinite set of imaginary black-hole resonances $\{\omega_n\}_{n=0}^{n=\infty}$ [7–13].

In a very interesting numerical investigation of the coupled Einstein-Yang-Mills equations, it was recently revealed by Rinne [14] that these unstable magnetically charged black-hole solutions play the role of approximate[1] codimension-two intermediate attractors (critical solutions[2]) in the nonlinear gravitational collapse of the Yang-Mills field [14, 16–19].[3] In particular, it has been shown explicitly [14] that the time spent in the vicinity of an unstable magnetically charged SU(2) Reissner-Nordström black-hole spacetime during a near-critical gravitational collapse of the nonlinear Yang-Mills field can be quantified by the characteristic scaling law[4]

$$\tau = \text{const} - \gamma \ln |p - p^*| \,. \tag{1.1}$$

[1]As discussed in [14], the SU(2) Reissner-Nordström black-hole spacetime is only an approximate intermediate attractor of the nonlinear gravitational collapse of the Yang-Mills field because it is characterized by an *infinite* family of exponentially growing (unstable) perturbation modes.

[2]See [15] for an excellent review on the black-hole critical phenomena in nonlinear gravitational collapse.

[3]It is worth noting that this physically interesting fact refers to type I and Type III nonlinear critical behaviors in gravitational collapse, see [14, 16–19] for details.

[4]Here the quantity $|p - p^*|$ provides a measure in the physical parameter space for the distance of the initial field data from the threshold (critical) solution of the nonlinear Einstein-Yang-Mills theory [15].

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It is interesting to note that the critical exponents in the scaling behavior (1.1), which characterizes the nonlinear near-critical gravitational collapse of the Yang-Mills field, are directly related to the imaginary eigenvalues which characterize the instability spectrum of the corresponding magnetically charged Reissner-Nordström black-hole spacetime [14]:

$$\gamma = 1/|\omega_{\text{instability}}|\,. \tag{1.2}$$

It is therefore physically interesting to investigate the characteristic instability (imaginary) resonance spectra $\{\omega_n(r_+,r_-)\}_{n=0}^{n=\infty}$ [5] of these magnetically charged black-hole solutions of the coupled nonlinear Einstein-Yang-Mills equations.

In his important numerical work, Rinne [14] has recently determined numerically the first three imaginary (unstable) resonant frequencies which characterize the magnetically charged SU(2) Reissner-Nordström black-hole spacetimes.[6] Subsequently, in [20] we have analyzed the detailed numerical data provided by Rinne [14] and revealed the intriguing fact that, to a good degree of accuracy, the numerically computed [14] excited instability eigenvalues of the magnetically charged SU(2) Reissner-Nordström black holes are characterized by the remarkably simple *universal* behavior

$$\omega_n(r_+-r_-)=\lambda_n \quad \text{for} \quad \omega_n r_+ \ll (r_+-r_-)/r_+\,, \tag{1.3}$$

where $\{\lambda_n\}$ are dimensionless constants which seem to be *independent* of the black-hole parameters.

The main goal of the present paper is to determine *analytically* the characteristic instability spectra of the magnetically charged SU(2) Reissner-Nordström black-hole solutions of the coupled Einstein-Yang-Mills theory. In particular, in this paper we shall provide a rigorous analytical proof for the validity of the numerically suggested [14, 20] universal behavior (1.3) which characterizes the excited instability spectra of the SU(2) Reissner-Nordström black-hole spacetimes.

2 Description of the system

The SU(2) Reissner-Nordström black-hole spacetime of mass M and unit magnetic charge is characterized by the spherically-symmetric line element [6]

$$ds^2=-\Big(1-\frac{2m}{r}\Big)dt^2+\Big(1-\frac{2m}{r}\Big)^{-1}dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2)\,, \tag{2.1}$$

where the radially dependent mass function $m=m(r)$ is given by[7]

$$m(r)=M-\frac{1}{2r}\,. \tag{2.2}$$

[5] Here $r_\pm$ are the black-hole horizon radii [see eq. (2.3) below].

[6] As noted above, the SU(2) Reissner-Nordström black-hole solutions of the coupled nonlinear Einstein-Yang-Mills field equations are characterized by *infinite* spectra $\{\omega_n(r_+,r_-)\}_{n=0}^{n=\infty}$ of imaginary (unstable) resonant frequencies [10, 11]. Reference [14] has provided, for the first time, detailed numerical results for the first three resonant frequencies which quantify the instability growth rates of these magnetically charged black-hole spacetimes.

[7] We shall use natural units in which $G=c=\hbar=1$.

The radii of the black-hole (outer and inner) horizons are given by

$$r_\pm = M \pm \sqrt{M^2-1}\,. \tag{2.3}$$

As shown in [21], linearized perturbation modes $\xi(r)e^{-i\omega t}$ [8] of the magnetically charged black-hole spacetime are governed by the wave equation

$$\Big[\frac{d^2}{dx^2}+\omega^2-U(x)\Big]\xi=0\,, \tag{2.4}$$

where the radial coordinate $x=x(r)$ is defined by the differential relation[9]

$$dx/dr=[1-2m(r)/r]^{-1}. \tag{2.5}$$

The effective radial potential which governs the Schrödinger-like wave equation (2.4) is given by [21]

$$U[x(r)]=-\frac{1}{r^2}\Big[1-\frac{2m(r)}{r}\Big]\,. \tag{2.6}$$

It is worth emphasizing the fact that the radial function $U(x)$ in eq. (2.4), which determines the spatial behavior of the black-hole perturbation modes, has the form of an effective binding potential. In particular, it is a negative definite function of the radial coordinate x and it vanishes asymptotically at the two boundaries $x\to\pm\infty$ of the magnetically charged black-hole spacetime. As shown in [21], well-behaved perturbation modes of the black-hole spacetime are characterized by spatially bounded (exponentially decaying) radial eigenfunctions at the two asymptotic boundaries:

$$\xi(x\to-\infty)\sim e^{|\omega|x}\to 0 \tag{2.7}$$

and

$$\xi(x\to\infty)\sim xe^{-|\omega|x}\to 0\,, \tag{2.8}$$

where $\omega=i|\omega|$.

The radial differential equation (2.4), supplemented by the physically motivated boundary conditions (2.7) and (2.8) [21], determine the discrete family $\{\omega_n(r_+,r_-)\}_{n=0}^{n=\infty}$ of unstable ($\Im\omega>0$) resonances which characterize the SU(2) Reissner-Nordström black-hole spacetimes [9–11]. Interestingly, below we shall show explicitly that the characteristic resonance spectrum of the magnetically charged black holes can be studied analytically in the regime $|\omega_n|r_+\ll 1$ of small imaginary resonant frequencies. In particular, we shall derive a remarkably compact *analytical* formula [see eq. (4.3) below] for the excited instability eigenvalues which characterize the SU(2) Reissner-Nordström black-hole solutions of the coupled Einstein-Yang-Mills theory.

[8]Note that unstable (exponentially growing in time) perturbation modes of the magnetically charged black-hole spacetime are characterized by the relation $\Im\omega>0$.

[9]Note that the near-horizon region $r\to r_+$ of the black-hole spacetime is mapped by the differential relation (2.5) to $x\to-\infty$, whereas spatial infinity $r\to\infty$ is mapped to $x\to\infty$.

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3 The characteristic resonance condition

The recent numerical results of Rinne [14] reveal that the excited instability resonances $\{\omega_n\}_{n=1}^{n=\infty}$ of the magnetically charged SU(2) Reissner-Nordström black-hole spacetimes are characterized by the property

$$|\omega_n|r_+ \ll 1\,; \qquad n = 1, 2, 3, \ldots \tag{3.1}$$

As we shall now show, the Schrödinger-like differential equation (2.4), which governs the dynamics of the black-hole perturbation modes, is amenable to an *analytical* treatment in the regime (3.1) of *small* resonant frequencies.

It was pointed out in [12, 13] that, in the $M \gg 1$ regime (the regime of weakly-magnetized SU(2) Reissner-Nordström black holes), the Schrödinger-like perturbation equation (2.4) can be transformed using the well-known Chandrasekhar transformations [22] to the physically equivalent Teukolsky-like radial equation [23]:

$$\Delta^2\frac{d^2\psi}{dr^2} + \big[\omega^2 r^4 + 2iM\omega r^2 - \Delta[2i\omega r + \ell(\ell+1)]\big]\psi = 0\,, \tag{3.2}$$

where the complex number

$$\ell \equiv \frac{-1+i\sqrt{3}}{2} \tag{3.3}$$

plays the role of an effective spherical harmonic index (see [12, 13] for details), and [23]

$$\Delta(r; M \gg 1) = r^2 - 2Mr\,. \tag{3.4}$$

The mathematical Chandrasekhar transformations [22] can also be used in the case of generic magnetically charged SU(2) Reissner-Nordström black holes,[10] in which case the generalized expression for the radial function $\Delta(r)$ in the Teukolsky-like radial perturbation equation (3.2) is given by [23]

$$\Delta(r; M) = r^2 - 2Mr + 1\,. \tag{3.5}$$

It is convenient to use the dimensionless physical variables [24–26]

$$z \equiv \frac{r - r_+}{r_+ - r_-}\,; \qquad k \equiv -i\omega(r_+ - r_-)\,; \qquad \varpi \equiv -i\omega\frac{2Mr_+}{r_+ - r_-}\,, \tag{3.6}$$

in terms of which the radial differential equation (3.2) reads

$$z^2(z+1)^2\frac{d^2\psi}{dz^2} + [-k^2z^4 + 2kz^3 - \ell(\ell+1)z(z+1) - \varpi(2z+1) - \varpi^2]\psi = 0\,. \tag{3.7}$$

[10] We use here the term 'generic SU(2) Reissner-Nordström black holes' to describe magnetically charged black holes with generic masses (that is, in the present analysis we do *not* assume the strong inequality $M \gg 1$ previously assumed in [12, 13]).

The physically acceptable solution[11] of the radial perturbation equation (3.7) in the near-horizon $kz \ll 1$ region[12] can be expressed in terms of the familiar hypergeometric function [24–27]:

$$\psi(z) = z^{1+\varpi}(z+1)^{1-\varpi}{}_2F_1(-\ell+1,\ell+2;2+2\varpi;-z)\,. \tag{3.8}$$

The physically acceptable solution[13] of the radial perturbation equation (3.7) in the asymptotic region $z \gg \varpi+1$ [14] can be expressed in terms of the familiar confluent hypergeometric function [24–27]:

$$\psi(z) = Ae^{kz}z^{\ell+1}{}_1F_1(\ell+2;2\ell+2;-2kz) + Be^{kz}z^{-\ell}{}_1F_1(-\ell+1;-2\ell;-2kz)\,. \tag{3.9}$$

For small resonant frequencies in the regime (3.1), the values of the dimensionless coefficients A and B in (3.9) can be determined by matching the two solutions [(3.8) and (3.9)] of the radial perturbation equation (3.7) in the overlap region[15]

$$\varpi + 1 \ll z \ll 1/k\,. \tag{3.10}$$

This matching procedure yields the expressions [24–26]

$$A = \frac{\Gamma(2\ell+1)\Gamma(2+2\varpi)}{\Gamma(\ell+2)\Gamma(\ell+1+2\varpi)} \tag{3.11}$$

and

$$B = \frac{\Gamma(-2\ell-1)\Gamma(2+2\varpi)}{\Gamma(-\ell+1)\Gamma(-\ell+2\varpi)} \tag{3.12}$$

for the dimensionless coefficients $\{A,B\}$ in (3.9).

Substituting the expressions (3.11) and (3.12) into the far-region expression (3.9) of the radial eigenfunction and using the asymptotic ($z \gg 1$) properties of the confluent hypergeometric functions [27], one finds the large-z asymptotic behavior [24–26]

$$\psi(z\to\infty) = \psi_1 re^{-kz} + \psi_2 r^{-1}e^{kz} \tag{3.13}$$

of the radial eigenfunctions, where

$$(r_+-r_-)\psi_1 = \frac{(2\ell+1)\Gamma^2(2\ell+1)\Gamma(2+2\varpi)}{\Gamma^2(\ell+2)\Gamma(\ell+1+2\varpi)}(-2k)^{-\ell} - \frac{(2\ell+1)\Gamma^2(-2\ell-1)\Gamma(2+2\varpi)}{\Gamma^2(-\ell+1)\Gamma(-\ell+2\varpi)}(-2k)^{\ell+1}, \tag{3.14}$$

[11] That is, the mathematical solution which respects the physically motivated near-horizon boundary condition (2.7).

[12] Note that in the near-horizon region $kz \ll 1$ one can neglect the first two terms inside the square brackets in eq. (3.7) [24].

[13] That is, the mathematical solution which respects the physically motivated asymptotic boundary condition (2.8) at spatial infinity.

[14] Note that in the asymptotic region $z \gg \varpi + 1$ one can neglect the last two terms inside the square brackets in eq. (3.7) [24].

[15] It is worth noting that the overlap radial region $\varpi \ll z \ll 1/k$ exists in the regime $|\omega|r_+ \ll 1$ of small resonant frequencies [see eqs. (3.1) and (3.6)].

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and

$$(r_+ - r_-)^{-1}\psi_2 = \frac{(2\ell+1)\Gamma^2(2\ell+1)\Gamma(2+2\varpi)}{\ell(\ell+1)\Gamma^2(\ell)\Gamma(\ell+1+2\varpi)}(2k)^{-\ell-2} - \frac{(2\ell+1)(\ell+1)\Gamma^2(-2\ell-1)\Gamma(2+2\varpi)}{\ell\Gamma^2(-\ell)\Gamma(-\ell+2\varpi)}(2k)^{\ell-1}. \tag{3.15}$$

A spatially bounded (normalizable) radial eigenfunction which satisfies the physically motivated boundary condition (2.8) at spatial infinity [21] is characterized by the asymptotic relation $\psi(z \to \infty) \to 0$. Thus, the coefficient ψ_2 of the exploding exponent in the asymptotic expression (3.13) should vanish, yielding the characteristic resonance equation [see eq. (3.15)]

$$(2k)^{2\ell+1} = \left[\frac{\Gamma(2\ell+1)\Gamma(-\ell)}{(\ell+1)\Gamma(-2\ell-1)\Gamma(\ell)}\right]^2 \frac{\Gamma(-\ell+2\varpi)}{\Gamma(\ell+1+2\varpi)} \tag{3.16}$$

for the instability eigenvalues of the SU(2) Reissner-Nordström black-hole spacetimes. Taking cognizance of eq. (3.3), one can write the resonance equation (3.16) for the instability spectra of the magnetically charged black holes in the form[16]

$$k^{i\sqrt{3}} = 8^{i\sqrt{3}} e^{i2\pi/3} \frac{\Gamma^2\big(i\frac{\sqrt{3}}{2}\big)}{\Gamma^2\big(-i\frac{\sqrt{3}}{2}\big)} \frac{\Gamma\big(\frac{1-i\sqrt{3}}{2}+2\varpi\big)}{\Gamma\big(\frac{1+i\sqrt{3}}{2}+2\varpi\big)}. \tag{3.17}$$

4 The black-hole excited instability spectra

As we shall now show, the characteristic resonance equation (3.17) for the instability eigenvalues of the magnetically charged SU(2) Reissner-Nordström black holes can be solved analytically in the regime[17]

$$\varpi \ll 1 . \tag{4.1}$$

In this small frequency regime the resonance equation (3.17) can be approximated by

$$k^{i\sqrt{3}} = 8^{i\sqrt{3}} e^{i2\pi/3} \frac{\Gamma^2\big(i\frac{\sqrt{3}}{2}\big)}{\Gamma^2\big(-i\frac{\sqrt{3}}{2}\big)} \frac{\Gamma\big(\frac{1-i\sqrt{3}}{2}\big)}{\Gamma\big(\frac{1+i\sqrt{3}}{2}\big)}, \tag{4.2}$$

which yields the characteristic *infinite* spectrum[18,19]

$$\omega_n \times (r_+ - r_-) = i \times 8e^{-\frac{2\pi}{\sqrt{3}}(n-\frac{1}{3})+\frac{4\theta-2\phi}{\sqrt{3}}}; \qquad n = 1, 2, 3, \ldots \tag{4.3}$$

of unstable ($\Im\omega > 0$) black-hole resonances, where

$$\theta \equiv \arg\big[\Gamma(i\sqrt{3}/2)\big]; \qquad \phi \equiv \arg\left[\Gamma\left(\frac{1+i\sqrt{3}}{2}\right)\right]. \tag{4.4}$$

[16] Here we have used eq. (6.1.18) of [27].

[17] Note that this regime corresponds to $\omega r_+ \ll (r_+ - r_-)/r_+$ [see eq. (3.6)].

[18] Here we have used the relation $1 = e^{-i2\pi n}$, where the integer $n = 1, 2, 3, \ldots$ is the resonance parameter of the black-hole perturbation mode. In addition, we have used here eq. (6.1.23) of [27].

[19] As noted in [12, 13], one finds the numerical ratio $\theta/2\phi = 1.0016$. Thus, one can replace, with an accuracy of 0.05%, the expression $(4\theta - 2\phi)/\sqrt{3}$ in (4.3) by the simpler term $\sqrt{3}\theta$.

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r_+	10.0	8.0	6.0	4.0	2.0	1.5
$\omega_2^{\text{ana}}/\omega_2^{\text{num}}$	1.046	1.025	1.011	1.005	1.007	1.012

Table 1. The excited instability eigenvalues of the magnetically charged SU(2) Reissner-Nordström black holes. We display the dimensionless ratio $\omega_2^{\text{ana}}/\omega_2^{\text{num}}$, where $\{\omega_2^{\text{ana}}(r_+)\}$ are the analytically calculated instability eigenvalues of the SU(2) Reissner-Nordström black holes as given by the analytical formula (4.3) and $\{\omega_2^{\text{num}}(r_+)\}$ are the corresponding numerically computed [14] instability eigenvalues of the magnetically charged black holes. One finds a fairly good agreement between the numerical data of [14] and the analytically derived formula (4.3) for the excited instability spectra of the SU(2) Reissner-Nordström black holes.

It is worth emphasizing again that the analytically derived formula (4.3) for the characteristic instability spectra of the magnetically charged SU(2) Reissner-Nordström black holes is valid in the small frequency regime [see (3.6) and (4.1)]

$$\omega_n r_+ \ll \frac{r_+ - r_-}{r_+} \, . \tag{4.5}$$

This inequality implies that, for a given value of the black-hole dimensionless temperature $(r_+ - r_-)/r_+$, the analytical formula (4.3) describes an infinite family of unstable (imaginary) black-hole resonances in the regime

$$n \gtrsim \left| \ln\left(\frac{r_+ - r_-}{r_+}\right) \right| + 1 \, . \tag{4.6}$$

It is interesting to note that the instability spectra (4.3) of the magnetically charged SU(2) Reissner-Nordström black-hole spacetimes have the simple generic form $\omega_n \times (r_+ - r_-) = \text{constant}_n \equiv \lambda_n$ [see eq. (1.3)]. We have therefore provided here an *analytical* proof for the *numerically*-observed [14, 20] universal behavior (1.3) of the black-hole excited instability eigenvalues.

5 Numerical confirmation

It is of considerable physical interest to verify the validity of the analytically derived formula (4.3) for the excited instability eigenvalues of the SU(2) Reissner-Nordström black holes. The corresponding instability eigenvalues of the magnetically charged black holes were recently computed numerically in the very interesting work of Rinne [14]. In table 1 we present the dimensionless ratio $\omega_2^{\text{ana}}/\omega_2^{\text{num}}$, where $\{\omega_2^{\text{ana}}(r_+)\}$ are the analytically calculated excited instability eigenvalues of the SU(2) Reissner-Nordström black holes as given by the analytical formula (4.3) and $\{\omega_2^{\text{num}}(r_+)\}$ are the numerically computed [14] instability eigenvalues of the magnetically charged black holes. From the data presented in table 1 one finds a fairly good agreement between the analytically derived formula (4.3) for the excited instability spectra of the SU(2) Reissner-Nordström black holes and the corresponding numerically computed black-hole instability eigenvalues.

6 Summary

The magnetically charged SU(2) Reissner-Nordström black-hole spacetimes describe a family of unstable solutions of the coupled nonlinear Einstein-Yang-Mills field equations [7–9, 12–14]. In particular, these magnetically charged black holes are known to be characterized by *infinite* spectra of imaginary (unstable) resonant frequencies $\{\omega_n(r_+, r_-)\}_{n=0}^{n=\infty}$. Based on direct *numerical* computations of the black-hole instability spectra [14], it has recently been pointed out [20] that the excited instability eigenvalues of the magnetically charged SU(2) Reissner-Nordström black holes are described, to a very good degree of accuracy, by the simple universal relation (1.3).

In the present paper we have studied analytically the characteristic instability spectra of the magnetically charged SU(2) Reissner-Nordström black-hole spacetimes in the small frequency regime. In particular, we have provided a simple *analytical* proof for the *numerically*-observed [14, 20] universal behavior [see eq. (1.3)]

$$\omega_n(r_+ - r_-) = \lambda_n \quad \text{for} \quad \omega_n r_+ \ll (r_+ - r_-)/r_+ \tag{6.1}$$

which characterizes the black-hole excited instability resonances, where $\{\lambda_n\}$ are constants. Our analysis has revealed that these dimensionless constants (which are *independent* of the black-hole parameters) are given by the simple relation [see eqs. (4.3) and (4.4)]

$$\lambda_n = i \times 8e^{-\frac{2\pi}{\sqrt{3}}(n-\frac{1}{3})+\frac{4\theta-2\phi}{\sqrt{3}}}. \tag{6.2}$$

Finally, it is interesting to note that one finds from (4.3) the simple dimensionless ratio

$$\frac{\omega_{n+1}}{\omega_n} = e^{-2\pi/\sqrt{3}} \tag{6.3}$$

for the characteristic excited instability eigenvalues of the magnetically charged SU(2) Reissner-Nordström black holes. It is worth emphasizing the fact that the relation (6.3), which characterizes the instability resonance spectra of the magnetically charged black-hole spacetimes, is *universal* in the sense that it is *independent* of the physical parameters (masses and magnetic charges) of the SU(2) Reissner-Nordström black holes.

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